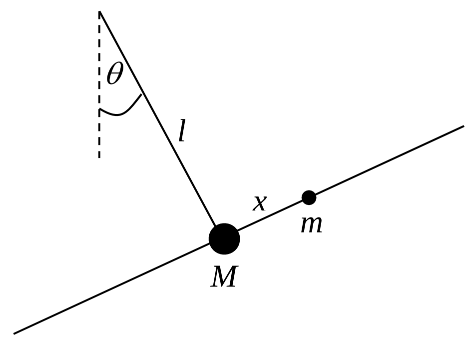
Ph.D. QUALIFYING EXAMINATION – PART A

Tuesday, January 14, 2014, 1:00 – 5:00 P.M.

Work each problem on a separate sheet(s) of paper and put your identifying number on each page. Do not use your name. Each problem has equal weight. A table of integrals can be used. Some physical constants and mathematical definitions will be provided if needed. Some students find useful the Schaum's outlines, 'Mathematical Handbook of Formulas and Tables'.

A1. A mass M is fixed at the intersection of two massless rods. The rods intersect at a right angle. The first rod has length l, and the second rod is very long. A second mass m is free to move without friction along the long rod (assume it can magically pass through mass M). The rod of length l is hinged at its support, and the entire system is free to rotate in the plane of the rods about the hinge. Let θ be the angle of rotation of the system and x be the distance between the two masses. Find the equations of motion.



A2. An uncharged metal (conducting) sphere of radius R is placed in an otherwise uniform electric field, $\vec{E} = E_0\,\hat{z}$. Recall that by symmetry there is no ϕ dependence, so the general solution to Laplace's equation for the potential is given as: $V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$.

a) Determine the potential $V(r,\theta)$ inside and outside the metal sphere.

b) Determine the electric field $\vec{E}(r,\theta)$ inside and outside the metal sphere.

c) Determine the surface charge density induced on the metal sphere.

d) Determine the electric dipole moment induced on the metal sphere.

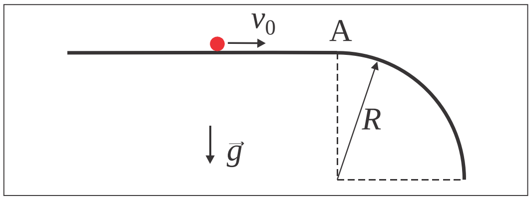
A3. A γ-ray photon of energy E_γ scatters elastically from an electron of mass m_e at rest. The photon is deflected in such a way that the angle between its new direction and the direction of the recoiled electron is 90 degrees. Find the energy of the scattered photon E'_γ and energy E'_e of the scattered electron.

A4. Consider 1-dimensional attractive delta function potential $V(x) = -\alpha\,\delta(x)$.

a) Calculate the bound state energy for a particle of mass m in the presence of this attractive delta function potential.

b) Calculate the transmission and reflection coefficients for a particle of mass m and energy E moving in the presence of this attractive delta function potential.

A5. A point mass m moves with initial speed v_0 along a frictionless track that consists of a horizontal part and a quarter circle of radius R (as shown in the figure).



a) What is the minimum value v_0^* of the initial speed such that the point mass loses contact with the track right where the horizontal part meets the quarter circle (point A)?

b) If the initial speed is below v_0^*, the point mass moves part way down the quarter circle before losing contact. Find the point where it loses contact as a function of v_0.

Hint: It may be advantageous to work in polar coordinates.

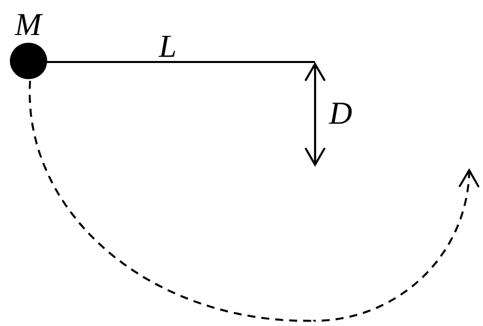
A6. A tritium nucleus (which contains two neutrons and a proton) is captured at a potential energy minimum associated with a vacancy in a crystal, where locally the electrostatic potential energy is quadratic, $U(x) = kx^2/2$. The captured nucleus undergoes β-decay by emitting a very fast electron, which converts it to a helium nucleus (with two protons and a neutron). As a result, it suddenly sees an effective spring constant k' that is twice the original value. If the tritium nucleus was initially in the vibrational ground state of the original potential, find the probability to find the new helium nucleus in the vibrational ground state of the new potential immediately after this happens. (Ignore the slight change in mass that occurs. Work in one dimension if you like, but explain how you would modify your answer for an isotropic three dimensional quadratic potential, $U(r)$.)

Ph.D. QUALIFYING EXAMINATION – PART B

Wednesday, January 15, 2014, 1:00 – 5:00 p.m.

Work each problem on a separate sheet(s) of paper and put your identifying number on each page. Do not use your name. Each problem has equal weight. A table of integrals can be used. Some physical constants and mathematical definitions will be provided if needed. Some students find useful the Schaum's outlines', 'Mathematical Handbook of Formulas and Tables'.

B1. A pendulum with mass M on a massless string of length L is released from rest from a horizontal position. At a distance D below the pivot for the pendulum is a nail. When the string hits the nail it causes the mass to move on an arc centered on the nail. Find the minimum distance D such that the mass will swing in a complete circle around the nail.



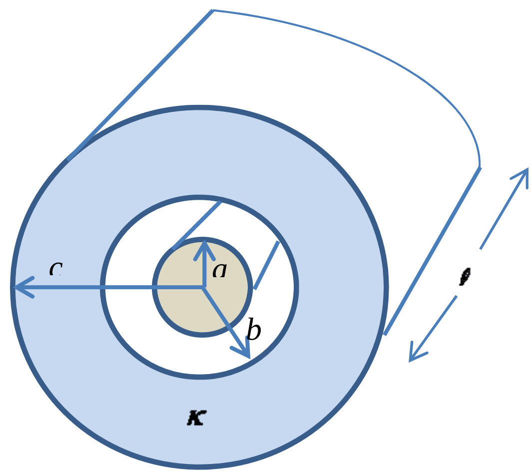
B2. A paramagnet can be viewed as a collection of N magnetic moments μ per unit volume that can be aligned by an external magnetic field.

(a) In the classical limit find an expression for the average value of the magnetic moment at temperature T and magnetic field B.

(b) Find an expression for the magnetic susceptibility χ at small B and show that it exhibits a $\frac{1}{T}$ dependence (Curie law).

B3. A particle is in a state associated with a wave-function of the form $\psi(\vec{r}) = e^{i\vec{q}\cdot\vec{r}}\,\phi(r)$, where $\phi(r)$ is a spherically-symmetric square-normalized wave function, and $\vec{q}$ is a fixed real wave-vector. Let $\langle T\rangle_\phi$ be the mean kinetic energy associated with the state $\phi(r)$. Find the mean position, mean momentum, and mean kinetic energy for a particle in the state $\psi(\vec{r})$. Express as a suitable integral over appropriate limits, the probability $P(r > a)$ that a position measurement will find the particle located outside a fixed radius a.

B4. A certain coaxial cable consists of a copper wire, radius a, surrounded by a thin copper tube of inner radius c. The space between is partially filled (from b out to c) with material of dielectric constant κ.



a) Determine the capacitance per unit length of this cable.

b) Determine the inductance per unit length of this cable.

c) If $b = 2a$, $c = 4a$ and $\kappa_r = 5$, what is the product of the capacitance per unit length times the inductance per unit length of this cable.

B5. A cubic box of linear size L contains N red particles of mass m_1 and N green particles of mass m_2. The particles can be considered classical non-interacting point masses. The system is in equilibrium at temperature T. A small hole of cross section A is made in one of the walls, causing particles to escape. Find the ratio between the numbers of red and green particles in the beam of escaping particles right after the hole is opened.

[Note: The system is macroscopic, i.e., $N \gg 1$ and $A \ll L^2$.]

B6. A thin, flat circular disc of radius R is located in the x–y plane with its center at the origin. The disc has a **uniform** charge density per unit area σ.

a) Find the potential everywhere along the positive z–axis.

b) Since there is no ϕ dependence, work in spherical polar coordinates (r,θ) so that the z-axis is described by $(r,0)$. Use your answer to part a) to find the potential above the disc and close to its center, correct through terms of order r^2. Recall the solution to Laplace's equation for the potential if there is no ϕ dependence is given as: $V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$.

c) From your answer to b) compute the radial component of the electric field in the plane of the disc, near its center, correct to terms of order r. Use the result to argue qualitatively how the charge would have to be distributed if the disc were conducting; would the charge pile up near the center or near the edge? Explain your reasoning.